44. a) \$1, 160%, compounded k times/yr for 1 year

$$\lim_{k\to\infty} 1\left(1+\frac{1}{k}\right)^{k\cdot 1} = e$$

$\cancel{1^\infty}$

$$\left(1+\frac{1}{10000}\right)^{10000} = 2.718\ldots$$

b) $y = y_0 e^{rt}$ $k = r = 1$

triple $3y_0 = y_0 e^{t}$ $\ln 3 = t$ 1.09

$3 = e^{t}$

c) $2.72\, y_0$

Dec 16-9:24 AM

24.

3 hr 10,000 – treat as start

5 hr 40,000

0 hr ? y_0

$y = y_0 e^{kt}$

$10{,}000 = y_0 e^{3k}$ ← solve for y_0 $y_0 = \frac{10000}{e^{3k}}$

$40000 = y_0 e^{5k}$ substitute

$40000 = \frac{10000}{e^{3k}} e^{5k} = 10000 e^{2k}$

$4 = e^{2k}$

$\ln 4 = 2k$ $T = \frac{\ln 2}{k}$

$\frac{\ln 4}{2} = k$

$10000 = y_0 e^{\frac{\ln 4}{2}\cdot 3}$

$y_0 = \frac{10{,}000}{e^{\frac{\ln 4}{2}\cdot 3}} = 1250$

Dec 16-9:35 AM

46. rule of 72 : double time $\approx \frac{72}{\text{rate}}$

$\frac{\ln 2}{k} = d.t.$

6% $dt = \frac{72}{6} = 12$ yrs

$\frac{100}{100} \cdot \frac{.69}{k} = \frac{69}{k\%} \to \frac{70}{k}$

rate $= \frac{72}{dt} = \frac{72}{5} \approx 14$

Dec 16-9:43 AM

6.4 newton's law of cooling

$\frac{dT}{dt} = -k(T - T_s)$

bigger

hotter

hot things cool faster

T_s = temp of surroundings

T = Temp of liquid

t = time

Dec 16-9:48 AM

Solve: $\frac{dT}{dt} = -k(T - T_s)$

$\int \frac{dT}{T - T_s} = \int -k \, dt$ sep. of var.

$e^{\ln|T - T_s|} = e^{(-kt + c)}$ integrated

drop abs. val. because $T > T_s$ so $T - T_s$ pos

$T - T_s = e^{-kt} \cdot e^{c} = ce^{-kt}$

initial cond. $t = 0$ $T = T_0$

$T_0 - T_s = C$

$T - T_s = (T_0 - T_s)e^{-kt}$

$$\boxed{T = T_s + (T_0 - T_s)e^{-kt}}$$

Dec 16-10:08 AM

Ex 6

egg $98^\circ C$ water 18°

$t = 5$ $T = 38^\circ$ — use to find k

$t = ?$ $T = 20^\circ$

Jake says: 1st find K

$T = T_s + (T_0 - T_s)e^{-kt}$

$38 = 18 + (98 - 18)e^{-k \cdot 5}$

$\frac{20}{80} = e^{-5k}$

$\ln(\frac{1}{4}) = -5k$

$k = \frac{\ln\frac{1}{4}}{-5} = \frac{\ln 1 - \ln 4}{-5} = \frac{\ln 4}{5}$

now find t

$20 = 18 + (98 - 18)e^{-\frac{\ln 4}{5}t}$ solve for t

$\frac{2}{80} = e^{-\frac{\ln 4}{5}t}$

$\ln\frac{1}{40} = -\frac{\ln 4}{5}t$

$t = \frac{\ln\frac{1}{40}}{-\ln 4} \cdot 5$

Dec 16-10:14 AM

Exp. decay

$$y = y_0 e^{-kt} \qquad H = \frac{\ln 2}{k}$$

$$k = \frac{\ln 2}{H}$$

$$y = y_0 e^{-\frac{\ln 2}{H} \cdot t}$$

$$y = y_0 \left(e^{-\ln 2}\right)^{\frac{t}{H}}$$

$$y = y_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$$

H = half life

Dec 16-10:23 AM

C 14 $\quad H = 5700$ yr

10% decayed means 90% of original present

$$y = y_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$$

solve for t

$$.90\,\cancel{y_0} = \cancel{y_0}\left(\frac{1}{2}\right)^{\frac{t}{5700}}$$

$$\ln .9 = t/5700 \ln(.5)$$

$$\ln .9 = \frac{t}{5700} \ln .5$$

$$t = \frac{5700 \ln .9}{\ln .5} \approx 866 \text{ yrs old}$$

Dec 16-10:27 AM

Dec 16-10:30 AM